INVESTIGATION OF CRACK OPENING IN ISOTROPIC STRAIN HARDENING MATERIAL

D. Pustaic[1)], M. Lovrenic-Jugovic[1)], H. Wolf[1)]

[1)] University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture, Ivana Lucica 5, 10 000 Zagreb, Croatia

ABSTRACT

The thin infinite plate with an embedded straight crack of a length 2a is considered. The plate is uniaxially loaded in its plane with uniformly distributed continuous load $\sigma_{yy}^{\infty} = \sigma_{\infty}$ in a direction perpendicular to the crack plane. The plate material is ductile so the small plastic zones around the crack tips are formed. Also, it is assumed that the plate material possesses a property of isotropic strain hardening. The strain hardening of the material is non-linear and it obeys the Ramberg-Osgood relation. The different stages of strain hardening of plate material are modeled by varying the parameters α and n in the Ramberg-Osgood's analytical expression. So, it is assumed that the strain hardening exponent n takes the values n = 3, 5, 7, 10, 25 and ∞.

The stress intensity factor (SIF) of the cohesive stresses $K_{coh}(a + r_p)$ is determined by means of Green's functions method. The crack tip and crack centre opening, or any arbitrary point at the crack surface, is determined by means of the displacement field of the points lying on the crack surface. The problem is formulated fully exactly and its solution is looked for by means of commercial software *„Mathematica“*. If it is assumed an appearance of small plastic zone around crack tip then it is possible to find an exact analytical solution expressed in a form of special gamma-functions $\Gamma(x)$. If the assumptions about small plastic zone are not introduced, then it is possible to give the solution by means of special hypergeometric functions ${}_2F_1(\alpha,\beta;\gamma;z)$.

KEYWORDS

Crack opening, isotropic strain hardening material, small plastic zone, strain hardening exponent, cohesive stresses, stress intensity factor, method of Green´s functions, commercial software *„Mathematica“*, gamma-function, hypergeometric function

INTRODUCTION

The paper deals with an analysis of the displacements of the points lying on a surface of straight crack of a length 2a. By means of the calculated displacements the crack tip opening displacements (CTOD) and the crack centre opening displacements (CCOD) are determined. The crack is incorporated in a thin infinite plate and it is spreaded throughout whole thickness of a plate. The plate is loaded uniaxially in-plane with uniformly distributed continuous load $\sigma_{yy}^{\infty} = \sigma_{\infty}$ in a direction of y-axis, while the crack surface is free of loading. Because a plate is thin the plane stress state is assumed, i. e. $\sigma_{xx} = \sigma_{xx}(x,y)$, $\sigma_{yy} = \sigma_{yy}(x,y)$ and $\sigma_{xy} = \sigma_{xy}(x,y)$. There are two axes of symmetry in the plate, x and y. The shear stresses $\sigma_{xy}(x,y)$ at the axes of symmetry equals zero, i.e. $\sigma_{xy}(x,0)=0$ and $\sigma_{xy}(0,y)=0.$ This statement will have as a consequence that the normal stresses at those axes will be , at the same time, the principal stresses: $\sigma_{xx}(x,0),$ $\sigma_{yy}(x,0),$ $\sigma_{xx}(0,y)$ and $\sigma_{yy}(0,y).$ Similar conclusion is valid for a field of the displacements and it means that it must be: $u(0,y)=0,$ $v(0,y)\neq 0$ and $u(x,0)\neq 0,$ $v(x,0)=0,$ but only for $x\geq a,$ while it is $v(x,0)\neq 0$ for $-a<x<a.$

As the plastic flow of a material occurs around the crack tips it means that the strain and stress fields will be elastic-plastic nature, exceptionally complex and it is very heavy to describe them exactly analytical. In this paper we will keep an assumption that those fields will have a structure of the HRR fields (Hutchinson, Rice, Rosengren). In a case of non-linear elastic plate material, similarly to isotropic and non-linear strain hardening material, those fields will have a structure, according to [4], [5]

$$u=r^{\lambda+1}\cdot\tilde{u}(\varphi),\qquad \varepsilon\sim r^{\lambda}\cdot\tilde{\varepsilon}(\varphi)\qquad \text{and}\qquad \sigma\sim r^{\lambda/n}\cdot\tilde{\sigma}(\varphi) \tag{1}$$

A rank of stress singularity within the crack tip depends on the parameter λ. The value of the parameter λ, for analyzing material, amounts $\lambda=-n/(n+1),$ so, for the structure of the mentioned fields is got

$$\sigma\sim r^{-1/(n+1)}\cdot\tilde{\sigma}(\varphi)\qquad \text{and}\qquad \varepsilon\sim r^{-n/(n+1)}\cdot\tilde{\varepsilon}(\varphi). \tag{2}$$

The cohesive stresses within a plastic zone around crack tip in isotropic strain hardening material will not be *constant* any more, but *variable,* and they will change according to some non-linear law. The non-linear distribution of the cohesive stresses around the crack tip must have the structure of the fields in accordance with the analytical expression (2). As the exact analytical solution is unknown, one of the possible approaches to the problem is the following. It is possible to determine the distribution of the cohesive stresses, for example, by the finite element method and then that distribution is approximated with an analytical expression, i. e. with some function, for example with an exponential function, or with a logarithmic function, or with a hyperbolic function and so on. The same approach was used in the paper [3] and it has shown very well. The authors M.Hoffman and T. Seeger have proposed in their paper [2] the next analytical expression

$$p(x)=\sigma_0\cdot\left[r_p/\left(x-a\right)\right]^{1/(n+1)} \tag{3}$$

for the distribution of the cohesive stresses. The quantity $p(x),$ in that expression, is a function of two parameters, i.e. *the magnitude of the plastic zone* around the crack tip r_p and *the strain hardening exponent* n. In the article [3] it has been shown that this expression approximates excellently the distribution of the cohesive stresses obtained by means of the finite element method. The same expression, the authors X. G. Chen, X. R. Wu and M. G. Yan have used in their paper [1].

In a frame of this paper, instead of the real physical elastic blunt crack, length of 2a and with a stress singularity within its tip, a fictitious elastic crack, the length of $2b=2(a+r_p)$ is observed. The normal stress $\sigma_{yy}(b,0)$ at the tip of that fictitious elastic crack has a final magnitude. The real, physical blunt crack and the plastic zone around its tip make the sharp fictitious elastic crack. A non-singularity stress condition within the tip of fictitious elastic crack is possible to write analytically in the following way

$$K(a+r_p)=K_{ext}(a+r_p)+K_{coh}(a+r_p)=0. \tag{4}$$

The singularity within the tip of a fictitious sharp elastic crack, $x=b=a+r_p,$ of the external load is canceled with the singularity of the cohesive stresses within the plastic zone.

CALCULATION OF DISPLACEMENTS FIELD AROUND THE CRACK TIP BY MEANS OF THE GREEN'S FUNCTIONS

On the basis of *Dugdale's model* the crack tip opening displacement (CTOD) can be determined on the following way

$$\delta_D = \delta_{ext} - \delta_{coh} = 2v_D, \tag{5}$$

where v_D denotes vertical displacement of crack tip. By means of *weight functions method (Green's functions)* the displacement v_D can be calculated as it follows

$$2v_D(x) = \frac{2}{E}\left[\int_x^b K_{ext}(\xi)\cdot m(\xi,x)d\xi - \int_{a_0}^b K_{coh}(\xi)\cdot m(\xi,x)d\xi\right], \tag{6}$$

where $m(\xi,x)$ is a Green's function of stress intensity factor (SIF) and it amounts

$$m(\xi,x) = 2\sqrt{\xi/\pi}\cdot(\xi^2 - x^2)^{-1/2}, \tag{7}$$

and $a_0 = a$ for $x < a$ and $a_0 = x$ for $x \geq a$. The first integral is easyly calculated by puting in $K_{ext}(\xi) = \sigma_\infty \cdot \sqrt{\pi\cdot\xi}$ and the expression (7). So, it is get

$$2v_{D.ext}(x) = \frac{2}{E}\int_x^b K_{ext}(\xi)\cdot m(\xi,x)d\xi = 4\cdot\frac{\sigma_\infty}{E}\sqrt{b^2 - x^2}. \tag{8}$$

The magnitudes of the displacements within the discrete points on the crack surface amount: $v_{D.ext}(0) = 2\sigma_\infty b/E,$ $v_{D.ext}(a) = 2\sigma_\infty\cdot\sqrt{b^2 - a^2}/E$ and $v_{D.ext}(b) = 0.$ The second integral in the expression (6) gives the displacements of the points lying on crack surface from cohesive stresses. The integral was solved in the paper [1] and it can be written

$$2v_{D.coh}(x) = \frac{2}{E}\int_{a_0}^b K_{coh}(\xi)\cdot m(\xi,x)d\xi = \frac{4}{\pi E}P(t)\cdot I(t)\Big|_a^b + \frac{8}{\pi E}\int_a^b P(t)\cdot\frac{t}{t^2 - x^2}\cdot\sqrt{\left(b^2 - x^2\right)/\left(b^2 - t^2\right)}\cdot dt. \tag{9}$$

Within the expression (9) the quantities P(t) and I(t) have the following meaning

$$P(t) = \int p(t)dt = \int \sigma_0\left[r_p/(t-a)\right]^{1/(n+1)}dt = \sigma_0\cdot r_p^{1/(n+1)}\cdot\left[(n+1)/n\right]\cdot(t-a)^{n/(n+1)}, \tag{10}$$

$$I(t) = \ln\left|\left(\sqrt{b^2 - x^2} + \sqrt{b^2 - t^2}\right)/\left(\sqrt{b^2 - x^2} - \sqrt{b^2 - t^2}\right)\right|. \tag{11}$$

For solving the integral (9) the substitution $t = b - r_p\cdot\xi$ was introduced which is analogous to substitution $x = b - r_p\cdot s.$ It is easyly to notice from the expression (10) that it is $P(a) = 0,$ and also, it can be seen, from the expression (11), that it is $I(b) = 0,$ so, it is possible to simplify the expression (9) and it looks like

$$2v_{D.coh}(x) = \delta_{coh}(x) = \frac{8}{\pi E}\int_a^b P(t)\cdot\frac{t}{t^2 - x^2}\cdot\sqrt{\left(b^2 - x^2\right)/\left(b^2 - t^2\right)}\cdot dt. \tag{12}$$

Including the right side of expression (10) under a sign of integral and transforming that expression introducing mentioned substitution it will be got

$$2v_{D.coh}(x)=\delta_{coh}(x)=\frac{8}{\pi}\cdot\frac{\sigma_0}{E}\int_1^0 r_p^{1/(n+1)}\cdot\frac{n+1}{n}\cdot r_p^{n/(n+1)}\cdot(1-\xi)^{n/(n+1)}\cdot$$

$$\cdot\frac{(b-r_p\xi):2br_p}{\left[2br_p\cdot(s-\xi)-r_p^2\cdot\left(s^2-\xi^2\right)\right]:2br_p}\cdot\frac{\sqrt{b^2-x^2}}{\sqrt{r_p\xi\left(2b-r_p\xi\right)}}\cdot\left(-r_p\cdot d\xi\right)= \tag{13}$$

$$=\frac{8}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\sqrt{b^2-x^2}\int_0^1\left(1-\xi\right)^{n/(n+1)}\cdot\frac{\frac{r_p}{2r_p}-\frac{r_p}{2b}\cdot\xi}{\left(s-\xi\right)-\frac{r_p}{2b}\left(s^2-\xi^2\right)}\cdot\frac{r_p}{\sqrt{r_p\xi\cdot 2b\left(1-\frac{r_p}{2b}\cdot\xi\right)}}\cdot d\xi.$$

Finally, if an assumption about *small plastic zone around the crack tip* (S.S.Y. condition) is introduced, then it can be taken $r_p/2b\approx 0$ and expression (13) assumes the form

$$2v_{D.coh}(x)=\delta_{coh}(x)=\frac{2\sqrt{2}}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\cdot\sqrt{r_p/b}\cdot\sqrt{b^2-x^2}\cdot\int_0^1\frac{(1-\xi)^{n/(n+1)}}{(s-\xi)\cdot\sqrt{\xi}}d\xi. \tag{14}$$

CRACK TIP OPENING DISPLACEMENT (CTOD) ASSUMING THE SMALL PLASTIC ZONE (S.S.Y.)

Presentation of exact analytical solution by means of special gamma-functions

Within the tip of real physical blunt crack a variable x amounts $x=a$. When it is included in the substitution $x=b-r_p\cdot s$ it follows that the variable s, within that point, takes the value 1, i.e. $s=1$. Now, the integral from the equation (14) takes the form

$$\int_0^1\frac{(1-\xi)^{n/(n+1)}}{\left(1-\xi\right)\cdot\sqrt{\xi}}d\xi=\int_0^1\left(1-\xi\right)^{\frac{n}{n+1}-1}\cdot\frac{1}{\sqrt{\xi}}\cdot d\xi=\int_0^1\left(1-\xi\right)^{-1/(n+1)}\cdot\xi^{-1/2}\cdot d\xi \tag{15}$$

and it can be solved analytical exactly. The solution of that type of integral will be

$$\int_0^1\xi^{\alpha}\cdot\left(1-\xi\right)^{\beta}\cdot d\xi=\Gamma\left(\alpha+1\right)\cdot\Gamma\left(\beta+1\right)/\Gamma\left(\alpha+\beta+2\right)=B\left(\alpha+1,\beta+1\right), \tag{16}$$

where $\Gamma(x)$ stands for *gamma-function* or *Euler's integral of second type,* while $B(x,y)=\Gamma(x)\cdot\Gamma(y)/\Gamma(x+y)$ denotes the *beta-function* or the *Euler's integral of first type.* Comparing the integrals within the expressions (15) and (16) we conclude that it is $\alpha=-1/2$ and $\beta=-1/(n+1)$. It is seen that $\alpha+1=1/2$ and $\beta+1=n/(n+1)$. Now, finally, by comparing with the expression (16), we get the solution of integral (15)

$$\int_0^1\xi^{-1/2}\cdot\left(1-\xi\right)^{-1/(n+1)}\cdot d\xi=\Gamma\left(1/2\right)\cdot\Gamma\left[n/\left(n+1\right)\right]/\Gamma\left[\left(1/2\right)+n/\left(n+1\right)\right]=B\left[1/2,\ n/\left(n+1\right)\right]. \tag{17}$$

It needs to be noted that the value of gamma-function amounts $\Gamma(1/2)=\sqrt{\pi}$. If so realized value of integral (15) is put in the equation (14), it is got an exact analytical expression for determination of crack tip opening displacement from the cohesive stresses, i.e.

$$2v_{D.coh}(a) = \delta_{coh}(a) = 2\sqrt{2/\pi}\cdot(\sigma_0/E)\cdot\left[(n+1)/n\right]\cdot\sqrt{r_p/b}\cdot\sqrt{b^2-a^2}\cdot \\ \cdot\Gamma\left[n/(n+1)\right]/\Gamma\left[(1/2)+n/(n+1)\right]. \tag{18}$$

Including the obtained solutions (8) and (18) in the relation (5) an analytical solution is got for crack tip opening displacement (CTOD). The gamma-functions appear within that solution and their argument depends on *strain hardening exponent* n. Furthermore, one more parameter r_p appears in that analytical solution which is calculated also by means of gamma-functions how it was shown within the papers [3], [4] and [5]. The crack tip opening displacement $\delta_D(a)$ was calculated on the basis of the solutions (8), (18) and (5) in depending of monotonously increasing external load of a plate σ_∞ and for the six different values of strain hardening exponent n = 3, 5, 7, 10, 25 and ∞. The diagram is, in dimensionless form, $\eta\delta_D(a)/a = f(\sigma_\infty/\sigma_0, n)$, shown on the Fig. 1. What kind of the influence has the strain hardening exponent n on the crack tip opening displacement $\delta_D(a)$ will be analysed at the chapter *Review on the obtained results.* Parameter η means $\eta = E/2\sigma_0$.

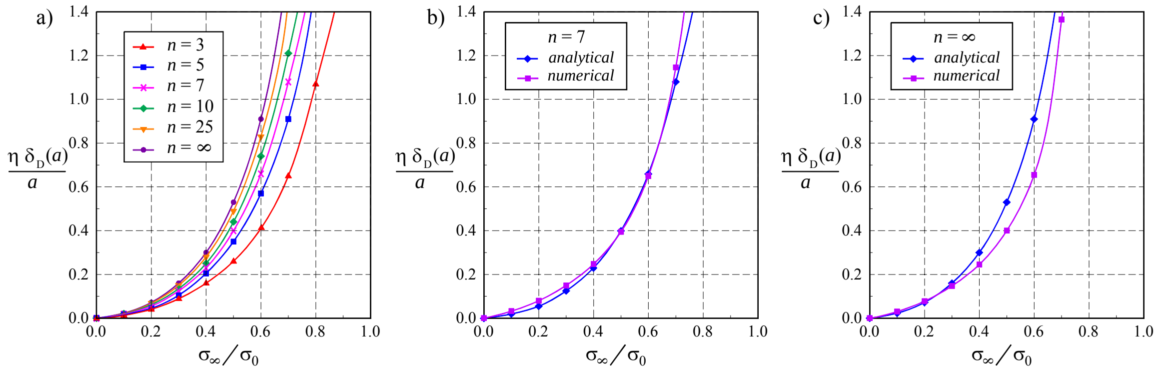

Fig. 1: Dependence of crack tip opening displacement $\eta\delta_D(a)/a$ on a monotonously increasing external load of a plate σ_∞/σ_0, for the different values of a strain hardening exponent n

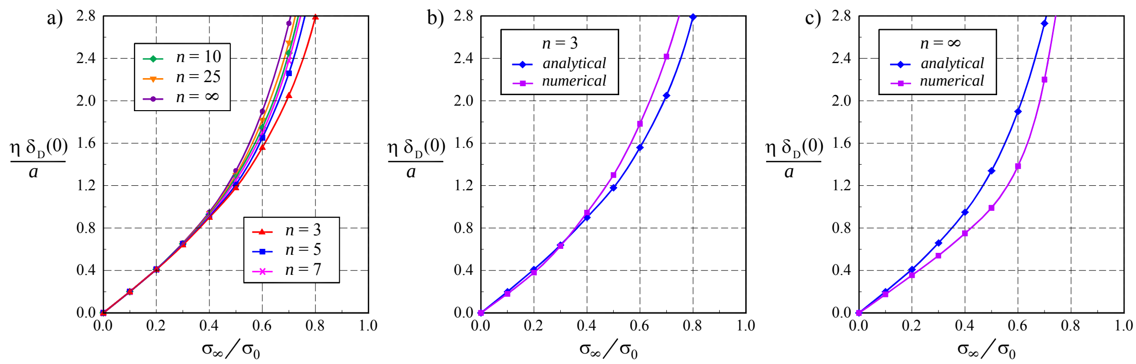

Fig. 2: Dependence of crack centre opening displacement $\eta\delta_D(x=0)/a$ on a monotonously increasing external load of a plate σ_∞/σ_0, for the different values of a strain hardening exponent n

CRACK CENTRE OPENING DISPLACEMENT (CCOD) BY THE ASSUMPTION OF SMALL PLASTIC ZONE AROUND THE CRACK TIP (S.S.Y.)

Within the crack centre, a variable x is equal to zero, i.e. $x = 0$. When it is included in the substitution $x = b - r_p \cdot s = 0$, we conclude that the variable s, at that point, takes the value $s = b/r_p$. As it always is $b > r_p$, $(b = a + r_p)$, we conclude that the variable s will be greater than one, i.e. $s > 1$, for $x = 0$. Now, the analytical expression (14) will be equal to

$$2v_{D.coh}(0) = \delta_{coh}(0) = \frac{2\sqrt{2}}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\cdot\sqrt{\frac{r_p}{b}}\cdot b\cdot\int_0^1\left\{\left[(1-\xi)^{n/(n+1)}\right]\Big/\left\{\left[\left(b/r_p\right)-\xi\right]\cdot\sqrt{\xi}\right\}\right\}d\xi =$$
$$= \frac{2\sqrt{2}}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\cdot\sqrt{\frac{r_p}{b}}\cdot b\cdot\int_0^1\left\{\left[(1-\xi)^{n/(n+1)}\right]\Big/\left\{\left(b/r_p\right)\cdot\left[1-\left(2r_p/2b\right)\cdot\xi\right]\cdot\sqrt{\xi}\right\}\right\}d\xi. \qquad (19)$$

Let us introduce the assumption about small plastic zone around the crack tips (S.S.Y.). In that case it could be taken that it is $r_p/2b \approx 0$ and that member within the denominator of sub integral expression is neglected. The expression (19), now, takes the form

$$2v_{D.coh}(0) = \delta_{coh}(0) = \frac{2\sqrt{2}}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\cdot\sqrt{\frac{r_p}{b}}\cdot r_p\cdot\int_0^1\frac{(1-\xi)^{n/(n+1)}}{\sqrt{\xi}}d\xi =$$
$$= \frac{2\sqrt{2}}{\pi}\cdot\frac{\sigma_0}{E}\cdot\frac{n+1}{n}\cdot\sqrt{\frac{r_p}{b}}\cdot r_p\cdot\int_0^1\xi^{-1/2}\cdot(1-\xi)^{n/(n+1)}d\xi. \qquad (20)$$

This type of integral is same as (15), or (16), respectively, only that the parameters α and β amount: $\alpha = -1/2$ and $\beta = n/(n+1)$, so it is $\alpha + 1 = 1/2$ and $\beta + 1 = (2n+1)/(n+1)$. Equally, it can be written that it is $\Gamma(\alpha+1+\beta+1) = \Gamma(\alpha+\beta+2) = \Gamma\left[(\alpha+1)+1+\beta\right] = \Gamma\left[(3/2)+n/(n+1)\right]$. Hence, the exact analytical solution for crack centre opening displacement (CCOD) from the cohesive stresses was given through special gamma-functions and it looks like

$$2v_{D.coh}(0) = \delta_{coh}(0) = 2\sqrt{2/\pi}\cdot(\sigma_0/E)\cdot\left[(n+1)/n\right]\cdot\sqrt{r_p/b}\cdot r_p\cdot$$
$$\cdot\Gamma\left[1+n/(n+1)\right]\Big/\Gamma\left[(3/2)+n/(n+1)\right]. \qquad (21)$$

Including the obtained solutions (8) and (21) in a relation (5), the analytical solution for the crack centre opening displacement (CCOD) is obtained. An argument of the special gamma-functions in the solution (21) depends on *strain hardening exponent* n. Within that solution there are two more physical quantities r_p and b. The magnitude of plastic zone around the crack tip r_p also depends on strain hardening exponent n and it, also, can be expressed through the special gamma-functions as it is shown within the papers [3], [4] and [5].

OPENING ANY ARBITRARY POINT AT CRACK SURFACE ASSUMING SMALL PLASTIC ZONE AROUND THE CRACK TIP (S.S.Y.)

Within the frame of non-linear fracture mechanics it is of interest to analyze the vertical displacements, or the crack opening, respectively, any arbitrary point at crack surface, i.e. for $0 < x < a$. This is especially important to know within an analysis of opening and closing of fatigue crack by its cyclic loading. In order to determine that, we start from the general expression (14) for crack opening from the cohesive stresses. Let us take the same substitution as previously, i.e. $x = a + r_p \cdot (1-\xi) = b - r_p\xi$ (analogously with $x = b - r_p \cdot s$). Consider the discrete point at crack surface, for example the one with $x = a/2$. Within that point the variable s takes the value $s = (b-x)/r_p = 1 + a/2r_p$. If we restrict to small plastic zone around the crack tip (S.S.Y. condition), it will be $r_p/a \le 1$. It is seen that the variable s will be greater than one $(s>1)$, for $r_p/a<1$, while for $r_p/a=1$, the variable s will take the value $s = 3/2$. We can conclude that then will be $a/2r_p \ge 1/2$. Under that condition the integral within the solution (14) can be transformed and shown in following form

$$\int_0^1 \left\{ \left[(1-\xi)^{n/(n+1)} \right] \Big/ \left[(s-\xi)\cdot\sqrt{\xi} \right] \right\} d\xi = \int_0^1 \left\{ \left[(1-\xi)^{n/(n+1)} \cdot \sqrt{\xi} \right] \Big/ \left[\xi \cdot \left(1 + \left(a/2r_p\right) - \xi\right) \right] \right\} \cdot d\xi. \qquad (22)$$

The solution of integral (22) was found by means of commercial software *„Mathematica"*, version 5.0, [9] and it amounts

$$\int_0^1 \left\{ \left[(1-\xi)^{n/(n+1)} \cdot \sqrt{\xi} \right] \Big/ \left[\xi \cdot \left(1 + \left(a/2r_p\right) - \xi\right) \right] \right\} \cdot d\xi = \left[\sqrt{\pi} \big/ \left(a + 2r_p\right) \right] \cdot \left\{ \Gamma\left[1 + n/(n+1)\right] \big/ \Gamma\left[(3/2) + n/(n+1)\right] \right\} \cdot$$
$$\cdot\, {}_2F_1\left[1/2,\ 1,\ (3/2) + n/(n+1),\ 2r_p \big/ \left(a+2r_p\right)\right], \qquad (23)$$

where ${}_2F_1\left[1/2,\ 1,\ (3/2) + n/(n+1),\ 2r_p \big/ \left(a+2r_p\right)\right]$ denotes *the hypergeometric function.* If the solution (23) is put in the expression (14), the final analytical expression for determination of opening any arbitrary point at crack surface $(x = a/2)$, from the cohesive stresses, is obtained. That expression will be

$$2v_{D.coh}(a/2) = \delta_{coh}(a/2) = \sqrt{2/\pi} \cdot (\sigma_0/E) \cdot \left[(n+1)/n\right] \cdot \sqrt{r_p/b} \cdot \sqrt{\left(3a + 2r_p\right) \big/ \left(a+2r_p\right)} \cdot$$
$$\cdot \left\{ \Gamma\left[1 + n/(n+1)\right] \big/ \Gamma\left[(3/2) + n/(n+1)\right] \right\} \cdot {}_2F_1\left[1/2,\ 1,\ (3/2) + n/(n+1),\ 2r_p \big/ \left(a+2r_p\right)\right]. \qquad (24)$$

By similar procedure can be determined the opening of any arbitrary point at the crack surface, $0 < x < a$. One should only notice that it is possible to change the quantity $(a + 2r_p)$ within the solution (23) with $2r_p s$ and $2r_p/(a+2r_p) = 1/s$. So, by means of the relations (14) and (23), it is got

$$2v_{D.coh}(x) = \delta_{coh}(x) = \sqrt{2/\pi} \cdot (\sigma_0/E) \cdot \left[(n+1)/n\right] \cdot \left(1/r_p\right) \cdot (1/s) \cdot \sqrt{r_p/b} \cdot \sqrt{b^2 - x^2} \cdot$$
$$\cdot \left\{ \Gamma\left[1 + n/(n+1)\right] \big/ \Gamma\left[(3/2) + n/(n+1)\right] \right\} \cdot {}_2F_1\left[1/2,\ 1,\ (3/2) + n/(n+1),\ 1/s\right]. \qquad (25)$$

The hypergeometric function ${}_2F_1\left[1/2,\ 1,\ (3/2) + n/(n+1),\ 1/s\right]$ is possible to develop in a series expansion as it follows

$${}_2F_1(\alpha,\ \beta, \gamma,\ z) = 1 + \frac{\alpha\beta z}{\gamma} + \frac{\alpha(1+\alpha)\beta(1+\beta)z^2}{2\gamma(1+\gamma)} + \frac{\alpha(1+\alpha)(2+\alpha)\beta(1+\beta)(2+\beta)z^3}{6\gamma(1+\gamma)(2+\gamma)} + O(z)^4. \qquad (26)$$

Under an assumption that the small plastic zone is formed around the crack tips (S.S.Y. condition) it is quite enough to take only first member of series expansion from the expression (26) and put it in the solutions (24) and (25), according to [5]. In that case the vertical displacements of the points lying on the crack surface will be also small.

REVIEW ON THE OBTAINED RESULTS

The aim of these investigations was to establish in what manner the isotropic strain hardening of a material influences on the magnitude of the displacements of the points lying on the crack surface, or on the crack opening displacements $\delta_D(x)$. An investigation of crack tip opening displacement (CTOD) and crack centre opening displacement (CCOD) are of special interest. By analysing the diagrams it is possible to conclude:

- isotropic strain hardening of a material will considerablly influences on crack opening displacement (COD) and how on its tip (CTOD) so on its centre (CCOD),
- that the crack tip opening displacement $\delta_D(a)$ will be as *bigger* as the strain hardening of a material is *smaller,* for the same level of external load. Therefore, *bigger* $\delta_D(a)$ for *bigger* strain hardening exponent n,
- that the crack tip opening displacement $\delta_D(a)$, for certain level of external load σ_∞, will be the largest by the *elastic-perfectly plastic* material $(n \to \infty)$,
- the crack centre opening displacement $\delta_D(x=0)$ is not sensitive to a level of isotropic strain hardening of a material (parameter n) at a low level of external load. Namely, it is clearly from the Fig. 2 that all the curves are coincided until approximately $\sigma_\infty / \sigma_0 = 0,4.$ That load approximately corresponds to the *limit load* at which the small plastic zone around the crack tip will be formed. At the bigger loads the crack centre opening displacement distinctly will depend on a level of isotropic strain hardening of a plate material (parameter n) and will be as *bigger* as the strain hardening of a material is *smaller.* Therefore, *bigger* $\delta_D(x=0)$ for *bigger* strain hardening exponent n,
- by anallysing the magnitude of the displacements of the points lying on the crack surface $2v_D(x)$, according to the solution (25) we conclude that the solution doesn't have to be *unambiguous* any more. It will depend on how many members of series expansion we include in a consideration when the hypergeometric function ${}_2F_1$ expands in a series, according to (26). To one certain external load of a plate σ_∞ it could correspond more different crack opening displacement (COD). The question is how much are those solutions precise and reliable? If it is taken only the first member of a series expansion the solution will be *unambiguous.*

CONCLUSION

In the analysis of crack opening displacement (COD) – (the parameter of Elastic Plastic Fracture Mechanics - EPFM), in this article, the analytical methods and the commercial software *„Mathematica“* were used. The displacements field around the crack tip in an elastic-plastic continuum, as from an external load of plate as well from the cohesive stresses was determined by means of *the method of the Green's functions*, expression (6). The stress intensity factor (SIF) of the cohesive stresses $K_{coh}(b)$ is calculated on the same way, [5]. The first and the second way require knowing the cohesive stresses distribution within a plastic zone. Exactly that is the biggest unknown. In this paper we have assumed that the cohesive stresses are distributed according to the analytical expression (3), [1], [2]. That expression turned out to be good and very precise approximation of a real stress distribution around the crack tip. The expression (3) has suffered the many criticisms of the scientific and researching circles. Neverthless, many authors have used it in their investigations, as for example [1], [4], [3], and so on.

Within the further investigations it will be suggested to measure the crack opening displacement (COD) and compare the results with the analytical solutions, in order to establish how precise and reliable are the results, we have obtained by analytical way. Also, it is planed, the investigations extend on the crack within the thick plate in which there are the triaxial stress state (state of plane strain) and to investigate what is the kind of influence of stress σ_{33} on the crack opening displacement (COD) and on the magnitude of plastic zone around the crack tip r_p.

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Corresponding author: dragan.pustaic@fsb.hr